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Indirect adaptive fuzzy control of nonlinear descriptor systems

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This paper focuses on indirect adaptive fuzzy control of nonlinear descriptor systems described by both uncertain algebraic and differential equations aiming to guarantee asymptotic tracking of a regular and impulse-free descriptor reference model. The proposed controller exploits the universal approximation capability of Takagi–Sugeno–Kang (TSK) fuzzy models for the identification of the unknown system dynamics. More specifically, it is assumed that only the system order is known while all the dynamical equations of the system are completely unknown. In the proposed method, the asymptotic tracking of the reference model is guaranteed by suitable adaptation laws for the parameters of the TSK fuzzy model. Simulation results are presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Descriptor systems described by both differential and algebraic equations have attracted considerable interest. For such systems there are still challenging control/estimation problems that need further investigation. Depending on the application domain, they are also known as singular, implicit, generalized, degenerated or differential-algebraic systems.

Compared to standard state-space systems, descriptor systems are able to describe more general real processes and can represent in a more accurate way the internal structure of certain physical systems such as electrical and analog mechanical circuits, aircraft dynamics, power systems and chemical processes. As a matter of fact, the standard state-space form has some weaknesses. One is that, in some practical models, both differential and algebraic equations may be used and the reduction to standard state-space form can be complicated or make the model lose some nice features. In particular, eliminating some variables for reduction to state-space form can lead to less meaningful variables in physical systems. Moreover, it has been shown in the literature that using descriptor models is more convenient and has higher capability to model large scale systems such as networks, power systems, etc. [23]. Dealing with descriptor systems actually requires more sophisticated analysis and design tools than for classical state-space systems. Further, for such systems, some well-known control approaches [7,18,19,31], have not yet been generalized.

A lot of research work has been carried out for extending classical state-space control approaches to descriptor systems [2,6,32,42,43]. In particular, optimal control was one of the first to be generalized to descriptor systems. Bender and Laub [5] developed an optimal linear-quadratic regulator (LQR) for continuous-time descriptor systems. Optimal control for linear descriptor systems and a quadratic cost functional has also been addressed by Razzaghi and Shafiee [26] exploiting Legendre series. In a recent study [33], optimal control is considered for an uncertain continuous-time descriptor system which can be described by uncertain differential equations; furthermore, uniqueness of the solution and stability in measure of the descriptor system are analysed. Robust control for descriptor systems has also been widely studied in the literature. Fang [12] proposed a state feedback controller based on linear matrix inequalities (LMIs) for the problem of delay-dependent robust H_∞-control of uncertain descriptor systems with time-delay. An LMI method to design state-space H_∞-controllers for linear time-invariant singular systems is proposed by Inoue et al. [16]. In [45], the mixed H_∞ and passive control problem is studied for descriptor systems with time-delay in order to make the considered system regular, impulse-free and stable. Further, other design schemes for descriptor systems have been developed such as a nonlinear feedback control law based on the problem of asymptotic output tracking for a class of nonlinear descriptor systems [44] and a nonlinear model-following control for fuzzy descriptor systems [36]. In [14], sliding mode control of discrete-time descriptor systems with external disturbances and time-varying delays is discussed.

Moreover, artificial intelligence techniques have received great attention in this area. For instance, a robust fuzzy controller has

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been designed to stabilize a class of singular systems in the presence of time-varying delay [41], and optimal control for a class of descriptor systems has been proposed exploiting a neural network to solve the matrix Riccati differential equation [3]. Li et al. present a sufficient condition for D-stability of a delayed discrete-time singular system to guarantee the regularity of the closed-loop system despite parameter uncertainties [24]. At the moment, only a few results are available on the control of uncertain descriptor systems. In particular, some adaptive control methods have been developed for descriptor systems. Azarfar et al. [27] have proposed an adaptive state feedback control approach based on a Lyapunov stability theorem. Another study concerns model-reference adaptive tracking of nonlinear descriptor systems with nonlinear parameterization. It converts this problem for a descriptor system into an equivalent problem for an ordinary state-space system [1]. Finally, an adaptive control scheme for a linear descriptor system has been developed [35] to ensure that the system states asymptotically track the reference states.

Recently, some interesting works have been devoted to control of systems with singular perturbations. In fact, *singularly perturbed systems* (SPSs) have attracted great attention due to their many relevant practical applications with physical systems displaying multiple time-scale features. Employing a *slow state-variable feedback* (SSVF) stabilization controller for semi-Markov jump discrete-time systems with slow sampling singular perturbations in [29], allowed to establish less conservative applicability conditions compared to existing studies. Shen et al. [28] have designed a controller for discrete-time nonlinear SPSs based on the *Takagi–Sugeno* (T–S) fuzzy model approach and semi-Markov kernel concept, and have analysed the stability of the proposed method. Quantized control for fuzzy *semi-Markov jump singularly perturbed systems* (S-MJSPSs) with packet dropouts is studied in [30] using T–S fuzzy modeling to cope with the system nonlinearity.

In nonlinear control theory, an important approach is to model the nonlinear system as T–S fuzzy systems. Consequently, it has attracted increasing attention. Stability analysis of T-S fuzzy systems is investigated using an equivalence relation and an extention of Polya's Theorem in [9]. Output feedback control of discrete-time T-S fuzzy systems is presented by Dong and Yong [8]. The proposed approach provides less conservative results since the premise variables of the fuzzy controller consist of both measurable premise variables and estimates of unmeasurable premise variables. A fault isolation scheme for T-S fuzzy systems with sensor faults is designed in [10] by exploiting premise variables in order to lead to improved fault isolation performance.

Another important approach is to approximate the unknown nonlinear dynamics of the system using fuzzy systems. Most of the mentioned methods about control of nonlinear descriptor systems assume that the plant dynamics are known. Nevertheless, in many practical systems the dynamics are not completely known. Since fuzzy systems are universal approximators [39], they can satisfactorily work in situations characterized by large uncertainty or unknown system dynamics. There exist a set of tunable parameters of the approximator fuzzy system that must be properly updated according to the system response. To this end, a suitable adaptation law can be used to update the parameters of the fuzzy system. Adaptive fuzzy control has recently attracted great interest in the context of ordinary systems [13,21,22,25,38] and the results show several advantages in controlling nonlinear systems having uncertainty or lacking information. Adaptive controllers can be classified into two categories, i.e. direct and indirect ones. In particular, *direct adaptive fuzzy control* (DAFC) uses a fuzzy system to generate the control action, and the parameters of the fuzzy system are directly adjusted to satisfy the required control objectives. Conversely, *indirect adaptive fuzzy control* (IAFC) exploits fuzzy systems in order to identify the plant dynamics, and a suitable controller is developed for the identified system. Both approaches have their own relative merits/drawbacks and can be applied to different problems. The main advantage of DAFC is that its structure is simpler. On the other hand, it is not important how many unknown functions are used to model the system, DAFC needs only as many fuzzy systems as the number of inputs. Further, in DAFC no prior knowledge about model functions may be needed but some restrictions exist in the design procedure. One advantage of IAFC is the separation of model adaptation from controller design, This enables model or parameter convergence to be analysed separately from control performance and stability. A further advantage is the ability to tune the desired control performance without the need for changing the fuzzy model and its parameters, since the fuzzy rules refer to the process and not to the controller. Clearly, changes in the controller do not imply any change in the plant model.

Since modelling of many physical systems naturally leads to consider both, possibly nonlinear, differential and algebraic equations and for such systems an ordinary state-space model is just a simplified model of the original one, adaptive fuzzy control should be extended to nonlinear descriptor systems. However, such an extension is not straightforward at all since the presence of algebraic equations brings complications in several issues of the control design/analysis such as model following conditions, stability analysis and so on.

Hence, the main motivations and contributions of this paper are the following.

- To find conditions which guarantee that a nonlinear descriptor system can track a desired linear descriptor reference model.
- To identify on-line a TSK fuzzy model that approximates the unknown nonlinear descriptor system.
- To propose an auxiliary signal that compensates the approximation error of the fuzzy model.
- To guarantee that the system state will asymptotically track the reference model state by means of the proposed indirect adaptive fuzzy controller.

As a consequence, this study investigates indirect adaptive fuzzy control for control-affine nonlinear descriptor systems. Fuzzy systems are employed to model the unknown nonlinear functions of the descriptor system and an adaptive estimation technique is used to on-line tune the fuzzy parameters. Furthermore, a compensating control is used to make amends for the estimation errors. Finally, stability of the closed-loop system and convergence of the system state to the desired one are proved by means of Lyapunov theory.

The rest of the paper is organized as follows. Section 2 provides the problem statement and some preliminaries. Then, Section 3 presents the indirect adaptive fuzzy control strategy. Simulation results are provided in Section 4. Section 5 concludes the paper.

2. wPreliminaries and statement of the problem

Consider the following descriptor control-affine system

$$\mathrm{E}\,\dot{\mathrm{x}} = \mathrm{f}(\mathrm{x}) + \mathrm{G}(\mathrm{x})\,\mathrm{u} \tag{1}$$

where: $\mathrm{x} \in \mathbb{R}^n$ is the state vector assumed to be accessible for feedback; $\mathrm{u} \in \mathbb{R}^{\mathrm{m}}$ is the control input; $\mathrm{f} : \mathbb{R}^{\mathrm{n}} \to \mathbb{R}^{\mathrm{n}}$ and $\mathrm{G} : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are unknown nonlinear function vector and matrix, respectively; $\mathrm{E} \in \mathbb{R}^{n\times n}$ is the known descriptor matrix with $rank(\mathrm{E}) = r \le n$. Let

$$\mathrm{f}(\mathrm{x}) = [f_1(\mathrm{x}), \ldots, f_n(\mathrm{x})]^T$$
$$\mathrm{G}(\mathrm{x}) = \begin{bmatrix} g_{11}(\mathrm{x}) & \ldots & g_{1m}(\mathrm{x}) \\ \vdots & \ddots & \vdots \\ g_{n1}(\mathrm{x}) & \ldots & g_{nm}(\mathrm{x}) \end{bmatrix} \tag{2}$$

where $f_i(x)$ and $g_{ij}(x)$ are unknown nonlinear functions.

The control objective is to find an indirect adaptive fuzzy controller such that the system state follow the state of the given reference model

$$\mathrm{E}\,\dot{\mathrm{x}}_\mathrm{d} = \mathrm{A}_\mathrm{d}\,\mathrm{x}_\mathrm{d} + \mathrm{B}_\mathrm{d}\,r \tag{3}$$

where: $\mathrm{x}_\mathrm{d} \in \mathbb{R}^\mathrm{n}$ is the desired state vector; A_d and B_d are known constant matrices; r is the reference model input. Notice that the descriptor matrix E in the above reference model will be assumed to be the same as in the original system (1).

Let us define the tracking error as

$$\mathrm{e} \triangleq \mathrm{x} - \mathrm{x}_\mathrm{d} \tag{4}$$

Then, the desired error dynamics is proposed to be

$$\mathrm{E}\,\dot{\mathrm{e}} = \mathrm{A}_\mathrm{d}\,\mathrm{e} \tag{5}$$

Definition 1 [23]**.** Consider a general linear descriptor system defined as

$$\mathrm{E}\dot{\mathrm{x}} = \mathrm{A}\mathrm{x} \tag{6}$$

- The matrix pair (E, A) is regular if $\det(s\mathrm{E} - \mathrm{A}) \neq 0$ for some $s \in \mathbb{C}$.
- The matrix pair (E, A) is impulse-free if $\deg(\det(s\mathrm{E} - \mathrm{A})) = rank(\mathrm{E})$.
- The matrix pair (E, A) is stable if all roots of $\det(s\mathrm{E} - \mathrm{A}) = 0$ are in the open left half plane.

Definition 2. Let $\mathrm{B}(x,t) \in \mathbb{R}^{n\times m}$ be a full rank matrix. Then, its pseudo-inverse $\mathrm{B}^\dagger(x,t) \in \mathbb{R}^{m\times n}$ is defined as

$$\mathrm{B}^\dagger(x,t) = \begin{cases} \mathrm{B}^T(x,t)\big(\mathrm{B}(x,t)\mathrm{B}^T(x,t)\big)^{-1}, & n < m \\ \mathrm{B}^{-1}(x,t), & n = m \\ \big(\mathrm{B}^T(x,t)\mathrm{B}(x,t)\big)^{-1}\mathrm{B}^T(x,t), & n > m \end{cases}.$$

Theorem 1 [17]**.** *For the descriptor system (6), the following statements are equivalent:*

- *The system* (E, A) *is regular, impulse-free and stable.*
- *There exists a solution* $\mathrm{P} \in \mathbb{R}^{n\times n}$ *to the following system of inequalities:*

$$\begin{cases} \mathrm{E}^T\mathrm{P} = \mathrm{P}^T\mathrm{E} \geq 0 \\ \mathrm{P}^T\mathrm{A} + \mathrm{A}^T\mathrm{P} = -\mathrm{Q} < 0. \end{cases} \tag{7}$$

Theorem 2. *A nonlinear descriptor system (1) can track the linear descriptor reference model (3), if the following conditions are satisfied*

$$\begin{aligned} &\mathcal{R}(\mathrm{G}(\mathrm{x})) \supset \mathcal{R}(\mathrm{B}_\mathrm{d}) \quad \forall \mathrm{x} \\ &\mathcal{R}(\mathrm{G}(\mathrm{x})) \supset \mathrm{Im}(\mathrm{h}) \quad \forall \mathrm{x} \end{aligned} \tag{8}$$

where: $\mathrm{h}(\mathrm{x}) \triangleq \mathrm{f}(\mathrm{x}) - \mathrm{A}_\mathrm{d}\mathrm{x}$; *Im*(h) *denotes the image of function* h; $\mathcal{R}(\mathrm{M})$ *stands for the range space of matrix* M.

Proof. Combining (1), (3) and (4) leads to

$$\mathrm{E}\dot{\mathrm{e}} = \mathrm{A}_\mathrm{d}\mathrm{e} + \mathrm{f}(\mathrm{x}) + \mathrm{G}(\mathrm{x})\,\mathrm{u} - \mathrm{B}_\mathrm{d}r - \mathrm{A}_\mathrm{d}\mathrm{x} \tag{9}$$

By defining

$$\mathrm{u} = \mathrm{G}^\dagger(\mathrm{x})(-\mathrm{f}(\mathrm{x}) + \mathrm{B}_\mathrm{d}r + \mathrm{A}_\mathrm{d}\mathrm{x}) \tag{10}$$

then (9) becomes

$$\mathrm{E}\dot{\mathrm{e}} = \mathrm{A}_\mathrm{d}\mathrm{e} + \big(\mathrm{G}(\mathrm{x})\mathrm{G}^\dagger(\mathrm{x}) - \mathrm{I}\big)(-\mathrm{f}(\mathrm{x}) + \mathrm{B}_\mathrm{d}r + \mathrm{A}_\mathrm{d}\mathrm{x}) \tag{11}$$

In order to satisfy the tracking condition, it is necessary to make (11) equal to (5). Hence, the following equality must hold

$$\big(\mathrm{G}(\mathrm{x})\mathrm{G}^\dagger(\mathrm{x}) - \mathrm{I}\big)(-\mathrm{f}(\mathrm{x}) + \mathrm{B}_\mathrm{d}r + \mathrm{A}_\mathrm{d}\mathrm{x}) = 0 \tag{12}$$

Since $\mathrm{G}(\mathrm{x})\mathrm{G}^\dagger(\mathrm{x})$ is an orthogonal projection operator on $\mathcal{R}(\mathrm{G}(\mathrm{x}))$ [11], it can be concluded that $\mathrm{G}(\mathrm{x})\mathrm{G}^\dagger(\mathrm{x}) - \mathrm{I}$ is also an orthogonal projection. Then,

$$\big(\mathrm{G}(\mathrm{x})\mathrm{G}^\dagger(\mathrm{x}) - \mathrm{I}\big)\mathrm{G}(\mathrm{x}) = 0. \tag{13}$$

Using (8) and (13), we have

$$\big(\mathrm{G}(\mathrm{x})\mathrm{G}^\dagger(\mathrm{x}) - \mathrm{I}\big)\mathrm{B}_\mathrm{d}r = 0$$

and

$$\big(\mathrm{G}(\mathrm{x})\mathrm{G}^\dagger(\mathrm{x}) - \mathrm{I}\big)(\mathrm{f}(\mathrm{x}) - \mathrm{A}_\mathrm{d}(\mathrm{x})) = 0$$

so that, if (8) are satisfied, (12) holds. □

Hereafter, it will be assumed that the reference model (3) satisfies the conditions stated in Theorems 1 and 2.

3. Main results

Consider the control-affine descriptor system (1). If f(x) and G(x) are known, under conditions (8), (5) can be deduced and, therefore, the tracking error converges to zero asymptotically. Whenever f(x) and G(x) are unknown, exploiting the fact that fuzzy models are universal approximators [40], they can be approximated by fuzzy models $\hat{\mathrm{f}}(\mathrm{x})$ and $\hat{\mathrm{G}}(\mathrm{x})$. Thus, the fuzzy counterpart of control law (41) turns out to be

$$\mathrm{u} = \hat{\mathrm{G}}^\dagger(\mathrm{x})(-\hat{\mathrm{f}}(\mathrm{x}) + \mathrm{B}_\mathrm{d}r + \mathrm{A}_\mathrm{d}\mathrm{x} + \mathrm{u}_\mathrm{s}) \tag{14}$$

where: $\hat{\mathrm{G}}^\dagger = (\hat{\mathrm{G}}^T\hat{\mathrm{G}} + \delta\mathrm{I})^{-1}\hat{\mathrm{G}}^T$ is a regularized pseudo-inverse of $\hat{\mathrm{G}}$, δ is a small positive constant; $\mathrm{u}_\mathrm{s} \in \mathbb{R}^\mathrm{n}$ is an auxiliary signal to be properly designed to compensate the unavoidable fuzzy approximation errors.

Substituting (14) into (9) yields

$$\begin{aligned} \mathrm{E}\dot{\mathrm{e}} = {} & \mathrm{A}_\mathrm{d}\mathrm{e} + (\mathrm{f}(\mathrm{x}) - \hat{\mathrm{f}}(\mathrm{x})) + (\mathrm{G}(\mathrm{x}) - \hat{\mathrm{G}}(\mathrm{x}))\mathrm{u} \\ & + \mathrm{u}_\mathrm{s} + (\hat{\mathrm{G}}(\mathrm{x})\hat{\mathrm{G}}^\dagger(\mathrm{x}) - \mathrm{I})(-\hat{\mathrm{f}}(\mathrm{x}) + \mathrm{B}_\mathrm{d}r + \mathrm{A}_\mathrm{d}\mathrm{x} + \mathrm{u}_\mathrm{s}) \end{aligned}$$

and defining

$$w_1 = (\hat{\mathrm{G}}(\mathrm{x})\hat{\mathrm{G}}^\dagger(\mathrm{x}) - \mathrm{I})(-\hat{\mathrm{f}}(\mathrm{x}) + \mathrm{B}_\mathrm{d}r + \mathrm{A}_\mathrm{d}\mathrm{x} + \mathrm{u}_\mathrm{s})$$

leads to

$$\mathrm{E}\dot{\mathrm{e}} = \mathrm{A}_\mathrm{d}\mathrm{e} + (\mathrm{f}(\mathrm{x}) - \hat{\mathrm{f}}(\mathrm{x})) + (\mathrm{G}(\mathrm{x}) - \hat{\mathrm{G}}(\mathrm{x}))\mathrm{u} + \mathrm{u}_\mathrm{s} + w_1 \tag{15}$$

Let us now recall that for a generic scalar function f(x), $\mathrm{x} \in \mathbb{R}^\mathrm{n}$, its fuzzy approximation $\hat{\mathrm{f}}(\mathrm{x})$ is obtained as the output of a fuzzy logic system based on the following subsection.

3.1. Takagi–Sugeno–Kang fuzzy structure

The type-1 TSK Multi-input Single-Output (MISO) model is defined as follows.

Definition 3. The TSK fuzzy system has the following IF-THEN rule structure [34]

$$\begin{aligned} &\mathrm{IF}\big(x_1 \text{ is } F_1^l\big)\text{and } \cdots \text{and } \big(x_n \text{ is } F_n^l\big)\mathrm{THEN} \\ &\quad y^l = c_0^l + c_1^l x_1 + \cdots + c_n^l x_n \end{aligned}$$

where: $l = 1, \ldots, L$, L being the number of fuzzy rules; $\mathrm{x} = [x_1, \ \ldots, \ x_n]^\mathrm{T} \in \mathbb{R}^\mathrm{n}$ is the input of the fuzzy system; y^l is the output of the local model in the rule consequent and c_i^l are real-valued parameters; F_i^l are fuzzy sets in $\mathbb{R}$ associated with the fuzzy membership functions $\mu_{F_i^l}(\cdot)$.

In the zero-order TSK system it is assumed that $c_i^l = 0$ for $i \neq 0$ and the unique remaining parameter is set to $c_0^l = \bar{y}^l$. Then, by using the singleton fuzzifier, the product inference engine, and a weighted-average [39], the final output of the fuzzy logic system can be expressed as

$$\hat{f}(x) = \frac{\sum_{l=1}^{L} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{L}\left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}$$

or, equivalently, as

$$\hat{f}(x) = \xi(x)\theta \tag{16}$$

where $\theta = \left[\bar{y}^1, \ldots, \bar{y}^L\right]^T$ is the parameter vector and $\xi(x) = \left[\xi^1(x), \ldots, \xi^L(x)\right]$ the vector of the fuzzy basis functions

$$\xi^l(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{L}\left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}$$

Theorem 3 [39]. *Let $f(x)$ be a continuous function defined on a compact set Ω. Then, for any given $\varepsilon > 0$ there exists a fuzzy logic system (16) such that*

$$\sup_{x\in\Omega} |f(x) - \xi(x)\theta| \le \varepsilon.$$

3.2. Fuzzy system design

According to the previous subsection, the fuzzy approximations of (2) take the form

$$\hat{f}_i(x|\theta_{f_i}) = \xi_{f_i}(x)\,\theta_{f_i} \qquad i = 1, \ldots, n$$
$$\hat{g}_{ij}(x|\theta_{g_{ij}}) = \xi_{g_{ij}}(x)\,\theta_{g_{ij}} \qquad i = 1, \ldots, n \qquad j = 1, \ldots, m$$

where: $\theta_{f_i} \in \mathbb{R}^{L_{f_i}}$ and $\theta_{g_{ij}} \in \mathbb{R}^{L_{g_{ij}}}$ are the parameter vectors for each system function $f_i(\cdot)$ or $g_{ij}(\cdot)$; $\xi_{f_i}(\cdot)$ and $\xi_{g_{ij}}(\cdot)$ are fuzzy basis function vectors defined as

$$\xi_{f_i}(x) = \left[\xi_{f_i}^1(x), \ldots, \xi_{f_i}^{L_{f_i}}(x)\right]$$
$$\xi_{g_{ij}}(x) = \left[\xi_{g_{ij}}^1(x), \ldots, \xi_{g_{ij}}^{L_{g_{ij}}}(x)\right]$$

Let us introduce

$$\xi_f(x) = diag\left(\xi_{f_1}(x), \ldots, \xi_{f_n}(x)\right)$$
$$\xi_g(x) = diag(\xi_{g_1}(x), \ldots, \xi_{g_n}(x))$$

where

$$\xi_{g_i}(x) = [\xi_{g_{i1}}(x), \ldots, \xi_{g_{im}}(x)]$$

and

$$\theta_f = \left[\theta_{f_1}^T, \ldots, \theta_{f_n}^T\right]^T$$
$$\theta_g = \left[\theta_{g_1}^T, \ldots, \theta_{g_n}^T\right]^T$$

where

$$\theta_{g_k} = diag\left(\theta_{g_{k1}}, \quad \ldots, \quad \theta_{g_{km}}\right).$$

Hence, the fuzzy-approximated system functions can be compactly expressed as

$$\hat{f}(x) = \hat{f}\left(x|\theta_f\right) = \xi_f(x)\,\theta_f$$
$$\hat{G}(x) = \hat{G}(x|\theta_g) = \xi_g(x)\,\theta_g \tag{17}$$

Let the optimal parameters $\theta_{f_i}^*$ and $\theta_{g_{ij}}^*$ of the closest possible approximations be defined as

$$\theta_{f_i}^* = \arg\min_{\theta_{f_i}\in\Omega_{f_i}} \left\{\sup\left|\hat{f}_i\left(x|\theta_{f_i}\right) - f_i(x)\right|\right\}$$
$$\theta_{g_{ij}}^* = \arg\min_{\theta_{g_{ij}}\in\Omega_{g_{ij}}} \left\{\sup\left|\hat{g}_{ij}\left(x|\theta_{g_{ij}}\right) - g_{ij}(x)\right|\right\}$$

where Ω_{f_i} and $\Omega_{g_{ij}}$ are the compact sets of allowable controller parameters. Hence, the best approximations of f(x) and G(x) are

$$\hat{f}(x|\theta_f^*) = \left[\hat{f}_1(x|\theta_{f_1}^*), \quad \ldots, \quad \hat{f}_n(x|\theta_{f_n}^*)\right]^T$$
$$\hat{G}(x|\theta_g^*) = \begin{bmatrix} \hat{g}_{11}(x|\theta_{g_{11}}^*) & \ldots & \hat{g}_{1m}(x|\theta_{g_{11}}^*) \\ \vdots & \ddots & \vdots \\ \hat{g}_{n1}(x|\theta_{g_{n1}}^*) & \ldots & \hat{g}_{nm}(x|\theta_{g_{nm}}^*) \end{bmatrix}$$

respectively, and

$$w_2 = [\hat{f}(x|\theta_f^*) - f(x)] + [\hat{G}(x|\theta_g^*) - G(x)]u \tag{18}$$

is defined as the minimum approximation error. In some adaptive fuzzy control schemes, it is assumed that the approximation error is small and can be neglected [4,15] or is square integrable [20,37] to make the stability analysis valid. In this study, the auxiliary signal u_s is used to compensate this error.

3.3. Adaptive fuzzy control design

In order to meet the control objectives under the unknown system uncertainties, a Lyapunov-based adaptive fuzzy controller is proposed in this subsection.

Using (18), (15) can be rewritten as

$$E\dot{e} = A_d e + [\hat{f}(x|\theta_f^*) - \hat{f}(x|\theta_f)] + [\hat{G}(x|\theta_g^*) - \hat{G}(x|\theta_g)]u + u_s + w \tag{19}$$

where $w = w_1 + w_2$. Substituting (17) into (19) and after some manipulations, the tracking error dynamics of the closed-loop system is given by

$$E\dot{e} = A_d e + \xi_f(\theta_f^* - \theta_f) + \xi_g(\theta_g^* - \theta_g)u + u_s + w \tag{20}$$

Theorem 4. *For the system (1), the control law (14) and auxiliary compensating signal*

$$u_s = -k\,\mathrm{sgn}(e_p) \tag{21}$$

will ensure that the system state is bounded and the tracking error asymptotically tends to zero if the following parameter adaptation laws are adopted

$$\dot{\underset{f}{\theta}} = \gamma_f \xi_f^T e_p \tag{22}$$

$$\dot{\underset{g}{\theta}} = \Gamma_g \xi_g^T e_p u^T \tag{23}$$

$$\dot{k} = \gamma_k \sum_{i=1}^{n} |e_{p_i}| \tag{24}$$

where:

$$e_p = Pe; \tag{25}$$

$\mathrm{sgn}(e_p) = \left[\ \mathrm{sgn}(e_{p_1}), \quad \ldots, \quad \mathrm{sgn}(e_{p_n})\ \right]^T$*; e_{p_i} is the ith component of e_p; matrix P satisfies*

$$\begin{cases} E^T P = P^T E \ge 0 \\ P^T A_d + A_d^T P = -Q \end{cases} \tag{26}$$

for some positive definite matrix Q.

Proof. Let

$$\tilde{k} \triangleq k_m - k,\ \tilde{\theta}_f \triangleq \theta_f^* - \theta_f,\ \tilde{\theta}_g \triangleq \theta_g^* - \theta_g \tag{27}$$

where: $\|w\| \le k_m$; k_m is an unknown positive constant; k is an estimate of k_m. It should be mentioned that k_m is an artificial quantity only required for analytical purposes.

Let us define the following Lyapunov function candidate

$$V = \frac{1}{2}\mathrm{e}^T\mathrm{E}^T\mathrm{P}\mathrm{e} + \frac{1}{2\gamma_f}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{1}{2}tr\left(\tilde{\theta}_g^T\Gamma_g^{-1}\tilde{\theta}_g\right) + \frac{1}{2\gamma_k}\tilde{k}^2 \tag{28}$$

where: γ_f, γ_k are positive constants; Γ_g is a diagonal matrix with positive elements. In the proposed Lyapunov function, the first term is chosen to guarantee that the system state will asymptotically track the reference model state. Moreover, it ensures stability of the closed-loop system according to Theorem 1. Conversely, the second and third terms are used to obtain the adaptive laws for online tuning of the fuzzy model parameters that approximate the unknown nonlinear functions. Finally, the update law for the coefficients of the compensating controller $\mathrm{u_s}$ is derived from the last term of the candidate Lyapunov function

Using (27), (20) will become

$$\mathrm{E}\dot{\mathrm{e}} = \mathrm{A_d}\mathrm{e} + \xi_f\tilde{\theta}_f + \xi_g\tilde{\theta}_g\mathrm{u} + \mathrm{u_s} + w$$

Differentiating V with respect to t yields

$$\begin{aligned}\dot{V} &= \frac{1}{2}\mathrm{e}^T\left(\mathrm{P}^T\mathrm{A_d} + \mathrm{A_d^T}\mathrm{P}\right)\mathrm{e} + \mathrm{e}^T\mathrm{P}^Tw + \mathrm{e}^T\mathrm{P}^T\xi_f\tilde{\theta}_f + \mathrm{e}^T\mathrm{P}^T\xi_g\tilde{\theta}_g\mathrm{u} \\ &\quad + \mathrm{e}^T\mathrm{P}^T\mathrm{u_s} + \frac{1}{\gamma_f}\dot{\tilde{\theta}}_\mathrm{f}^T\tilde{\theta}_\mathrm{f} + \mathrm{tr}\left(\dot{\tilde{\theta}}_\mathrm{g}^T\Gamma_\mathrm{g}^{-1}\tilde{\theta}_\mathrm{g}\right) + \frac{1}{\gamma_\mathrm{k}}\dot{\tilde{k}}\tilde{k}\end{aligned} \tag{29}$$

The fact that the trace is invariant under cyclic permutations i.e. $tr(A,\ B,\ C,\ D) = tr(D, A, B, C)$ where A, B, C and D are matrices of appropriate dimensions, leads to

$$\mathrm{e}^T\mathrm{P}^T\xi_g\tilde{\theta}_g\mathrm{u} = tr(\mathrm{e}^T\mathrm{P}^T\xi_g\tilde{\theta}_g\mathrm{u}) = tr(\mathrm{u}\mathrm{e}^T\mathrm{P}^T\xi_g\tilde{\theta}_g) \tag{30}$$

Consequently, by using (27) and (30), (29) can be rewritten as

$$\begin{aligned}\dot{V} &= \frac{1}{2}\mathrm{e}^T\left(\mathrm{P}^T\mathrm{A_d} + \mathrm{A_d^T}\mathrm{P}\right)\mathrm{e} - \frac{1}{\gamma_f}\dot{\theta}_f^T\tilde{\theta}_f + \mathrm{e}^T\mathrm{P}^T\xi_f\tilde{\theta}_f \\ &\quad - tr(\dot{\theta}_g^T\Gamma_g^{-1}\tilde{\theta}_g) + tr(\mathrm{u}\mathrm{e}^T\mathrm{P}^T\xi_g\tilde{\theta}_g) + \mathrm{e}^T\mathrm{P}^T\mathrm{u_s} + \mathrm{e}^T\mathrm{P}^Tw + \frac{1}{\gamma_k}\dot{\tilde{k}}\tilde{k}\end{aligned} \tag{31}$$

Substituting (22), (23) and (26) into (31), $\dot{V}$ can be bounded as

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}\mathrm{e}^T\mathrm{Q}\mathrm{e} - \frac{1}{\gamma_f}(\gamma_f \mathrm{e}_p^T\xi_f)\tilde{\theta}_f + \mathrm{e}^T\mathrm{P}^T\xi_f\tilde{\theta}_f \\ &\quad + tr\left((\mathrm{u}\,\mathrm{e}_p^T\,\xi_g\,\Gamma_g)\Gamma_g^{-1}\tilde{\theta}_g\right) + tr(\mathrm{u}\mathrm{e}^T\mathrm{P}^T\xi_g\tilde{\theta}_g) \\ &\quad + \mathrm{e}^T\mathrm{P}^T\mathrm{u_s} + \mathrm{e}^T\mathrm{P}^Tw + \frac{1}{\gamma_k}\dot{\tilde{k}}\tilde{k}\end{aligned} \tag{32}$$

and results

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T\mathrm{Q}\mathrm{e} + \mathrm{e}^T\mathrm{P}^Tw + \mathrm{e}^T\mathrm{P}^T\mathrm{u_s} + \frac{1}{\gamma_k}\dot{\tilde{k}}\tilde{k} \tag{33}$$

Considering the definition (25) and

$$\mathrm{e}^T\mathrm{P}^Tw = \mathrm{e}_p^Tw \le \sum_{i=1}^{n}|w_i||e_{p_i}| \le k_m\sum_{i=1}^{n}|e_{p_i}|$$

the following inequality will be obtained

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T\mathrm{Q}\mathrm{e} + k_m\sum_{i=1}^{n}|e_{p_i}| - k\sum_{i=1}^{n}|e_{p_i}| + \frac{1}{\gamma_k}\dot{\tilde{k}}\tilde{k}$$

Substitution of the adaptation law (24) and (27) give

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T\mathrm{Q}\mathrm{e} \le -\frac{1}{2}\lambda_{\min}(\mathrm{Q})\|\mathrm{e}\|^2 \tag{34}$$

Integrating (34) over $[0, \infty)$, yields

$$\int_0^\infty \|\mathrm{e}\|^2 dt \le \frac{V(0) - V(\infty)}{\frac{1}{2}\lambda_{\min}(\mathrm{Q})} \tag{35}$$

The Lyapunov function candidate (28) is nonincreasing and lower bounded ($V \ge 0$). According to the inequality (35), $\|\mathrm{e}\|$ exists and is finite ($\mathrm{e}(t) \in L_2$). Moreover, from (28) it can be deduced that $\mathrm{e}(t) \in L_\infty$ and θ_f, θ_g, k are bounded. Further, from (20) it turns out that $\dot{\mathrm{e}}(t) \in L_\infty$, so that $\mathrm{e}(t)$ is uniformly continuous. Based on Barbalat's lemma it holds that $\lim_{t\to\infty}\mathrm{e}(t) = 0$. □

Remark 1. The compensating signal (21) causes a phenomenon so-called *chattering*, consisting of unwanted oscillations in the control and other system signals. Thus, usually a smooth function such as saturation or hyperbolic tangent is used instead of the sign function. In the simulations of next section, we have specifically used the saturation, instead of the sign function, to implement the compensating controller (21).

The overall structure of indirect adaptive fuzzy tracking control is schematized in Fig. 1.

4. Simulations

In this section, the proposed adaptive fuzzy controller is applied to tracking control of two nonlinear singular systems in order to investigate the effectiveness of this approach.

Example 1. Consider the following nonlinear singular system

$$\begin{aligned}\dot{x}_1 &= -3x_1 - 2x_2 \\ 0 &= x_2 + e^{x_2} + e^{x_1}u\end{aligned} \tag{36}$$

where $\mathrm{x} = [x_1, x_2]^T$ is the state vector and u the scalar control input which should be designed based on the scheme presented in this paper. The reference state is generated from (3) where

$$\mathrm{E} = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ \mathrm{A_d} = \begin{bmatrix}-3 & -2\\ -1 & 2\end{bmatrix},\ \mathrm{B_d} = \begin{bmatrix}0\\ 4\end{bmatrix} \tag{37}$$

and the input of the reference system is a pulse wave signal defined as

$$r(t) = \begin{cases}1, & t \le 10\\ 0, & 10 < t \le 20\end{cases}$$

$$r(t+20) = r(t)$$

It can be checked that (36) and (37) satisfy the conditions in (8). Hence, the method discussed in this paper can be applied to this example. It is assumed that the initial state is set to $\mathrm{x}(0) = [0.5, -0.56]^T$ and the initial conditions of the fuzzy parameters are set to zero for all elements of $\boldsymbol{\theta}_f$ and to one for all elements of θ_g. Further, the learning parameters are chosen as $\gamma_f = 5$, $\Gamma_g = 0.5$ and $\gamma_k = 10$. The matrix P satisfying (26) has been chosen as

$$\mathrm{P} = \begin{bmatrix}0.156 & 0\\ 0.031 & -0.25\end{bmatrix}$$

To approximate the unknown nonlinear functions, five fuzzy membership functions in the interval [-1, 1] are considered for normalized x_i $(i = 1, 2)$

$$\mu^1(x_i) = \exp\left(-\frac{1}{2}(x_i - 1)^2\right)$$

$$\mu^2(x_i) = \exp\left(-\frac{1}{2}(x_i - 0.5)^2\right)$$

$$\mu^3(x_i) = \exp\left(-\frac{1}{2}(x_i)^2\right)$$

$$\mu^4(x_i) = \exp\left(-\frac{1}{2}(x_i + 0.5)^2\right)$$

$$\mu^5(x_i) = \exp\left(-\frac{1}{2}(x_i + 1)^2\right)$$

Figs. 2–4 display the simulation results using the controller (14) for the system (36). It can be seen, from Figs. 2 and 3, that a good

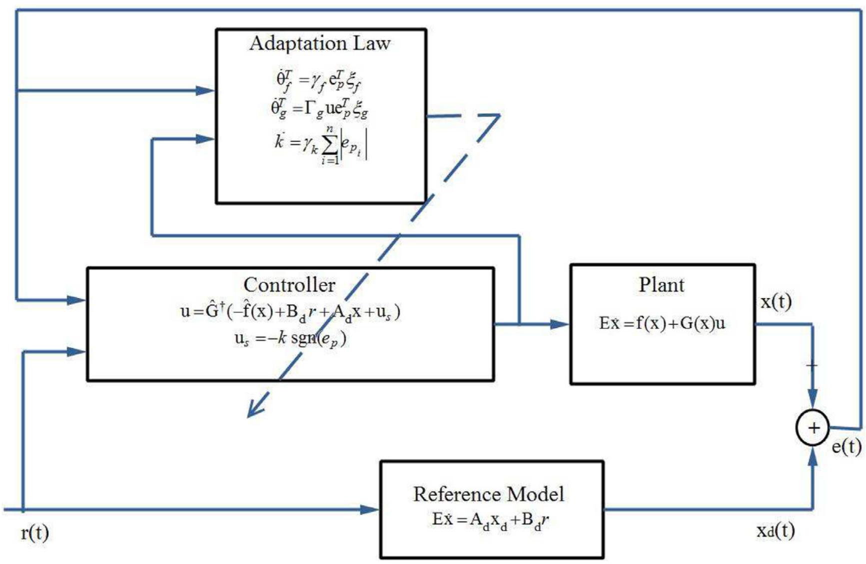

Fig. 1. Indirect adaptive fuzzy tracking control.

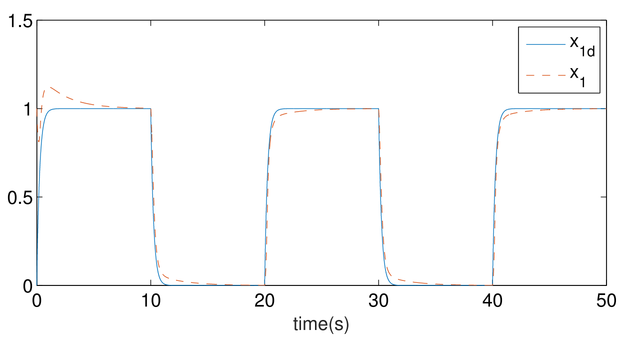

Fig. 2. Trajectory of the state variable x_1 and reference state variable x_{1d}

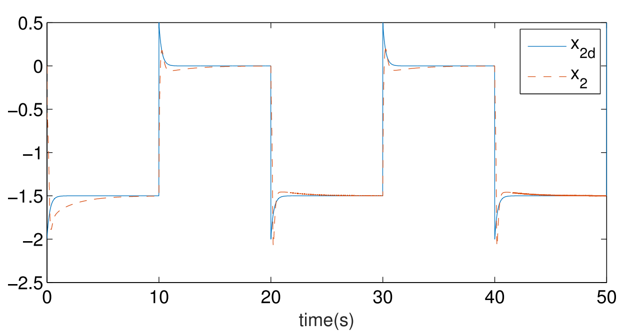

Fig. 3. Trajectory of the state variable x_2 and reference state variable x_{2d}.

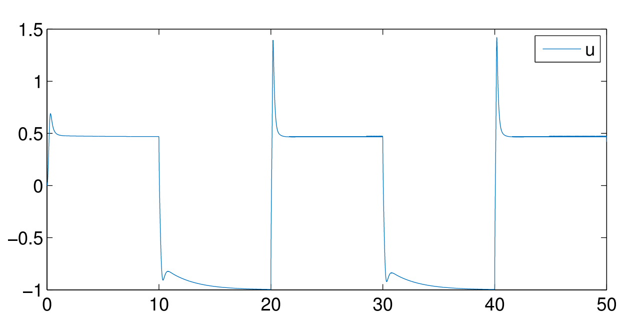

Fig. 4. Control input signal u.

tracking performance can be obtained under the action of the proposed controller. Fig. 4 shows the control input signal.

Example 2. Let us now consider the following control-affine descriptor system

$$\begin{aligned} \dot{x}_1 &= 2x_1^2 - 3x_1 - 3x_3 + e^{(x_1+x_3)}u_1 \\ \dot{x}_2 &= x_1x_2 - 2x_2 + (x_1+1)u_2 \\ 0 &= x_3 + \cos x_3 + x_2 + (\sin x_1 + 0.5\cos x_3)u_3 \end{aligned} \tag{38}$$

where $\mathrm{x} = [x_1, x_2, x_3]^T$ denotes the state and $\mathrm{u} = [u_1, u_2, u_3]^T$ the control input.

Let us also consider the reference descriptor model (3) with matrices

$$\mathrm{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{A_d} = \begin{bmatrix} -3 & 0 & -2 \\ 2 & -1 & 0 \\ -0.5 & 0 & 1 \end{bmatrix}, \quad \mathrm{B_d} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \tag{39}$$

and sinusoidal reference input

$$r(t) = 0.5\,\sin(0.1\pi t)$$

The fuzzy membership functions are defined, for any state variable x_i, as follows

$$\mu^1(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i - 1.25}{0.6}\right)^2\right)$$

$$\mu^2(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i}{0.6}\right)^2\right)$$

$$\mu^3(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i + 1.25}{0.6}\right)^2\right)$$

Then, there are 27 rules to approximate the system functions f(x) and G(x).

The initial state has been set to $\mathrm{x}(0) = [0.5, 0.5, -1.5]^T$ and the initial values of the fuzzy parameters $\boldsymbol{\theta}_f$ and θ_g have been set to one. The matrix P satisfying (26) has been chosen as

$$\mathrm{P} = \begin{bmatrix} 0.256 & 0.2 & 0 \\ 0.2 & 0.5 & 0 \\ 0.262 & 0.4 & -0.5 \end{bmatrix}$$

The adaptation gains used in this simulation are $\gamma_f = 1000$, $\Gamma_g = 50\,\mathrm{I}_3$ and $\gamma_k = 100$. It must be noted that Theorem 2 holds for any system in the form (1) with full rank square matrix G.

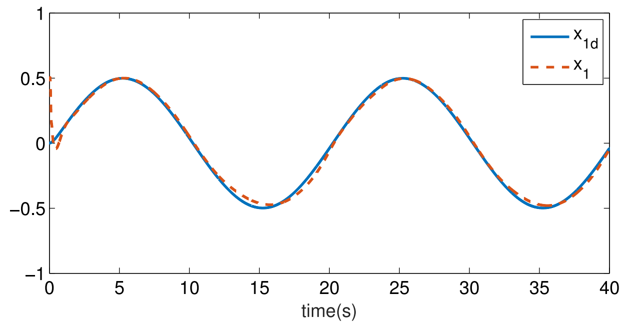

Fig. 5. Trajectory of the state variable x_1 and reference state variable x_{1d}.

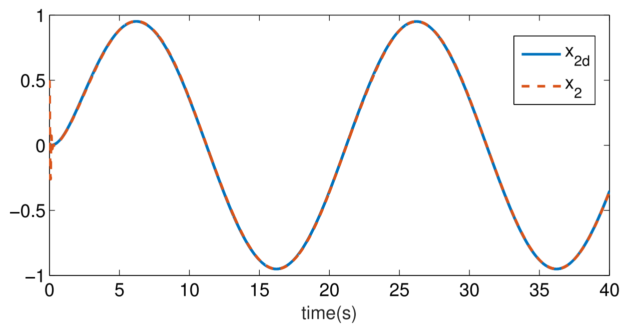

Fig. 6. Trajectory of the state variable x_2 and reference state variable x_{2d}.

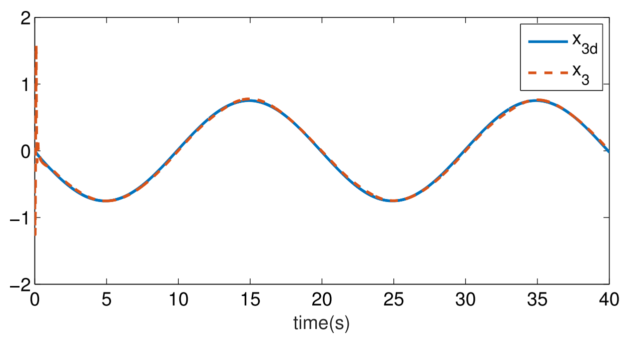

Fig. 7. Trajectory of the state variable x_3 and reference state variable x_{3d}.

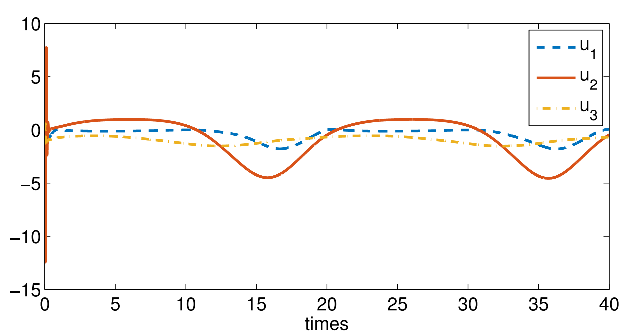

Fig. 8. Control input signals u.

The simulation results are shown in Figs. 5–8 where Figs. 5–7 show the tracking performance for x_1, x_2, x_3, respectively. It is clear that states rapidly converge to the desired reference trajectories. The control input signals are shown in Fig. 8.

The simulation results demonstrate the tracking capability of the proposed controller and its effectiveness for control of uncertain control-affine descriptor systems.

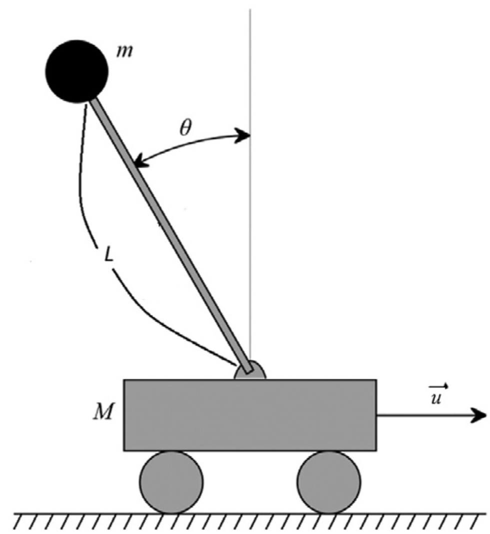

Fig. 9. An inverted pendulum system.

Example 3. In this example, we consider an inverted pendulum system consisting of a mass m connected, by means of a massless rod of length L, to a cart of mass M (see Fig. 9). The motion equation of such a system is given by Wang [39]

$$\ddot{\theta} - \left[g\sin\theta - \frac{mL\dot{\theta}^2\cos\theta\sin\theta}{M+m}\right] \Big/ \left[L\left(\frac{4}{3} - \frac{m\cos^2\theta}{M+m}\right)\right]$$
$$-\frac{\cos\theta}{M+m} \Big/ \left[l\left(\frac{4}{3} - \frac{m\cos^2\theta}{M+m}\right)\right]u = 0$$

Given the parameter values $M = 1[kg]$, $m = 0.1[kg]$, $L = 0.5[m]$, $g = 9.8[m/s^2]$ and defining the state vector $x = [x_1, x_2, x_3]^T = \left[\theta, \dot{\theta}, \ddot{\theta}\right]^T$, the resulting nonlinear descriptor system turns out to be:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ 0 &= 0.66x_3 - 0.045x_3\cos^2 x_1 - 9.8\sin x_1\\ &\quad + 0.045x_2^2\sin x_1\cos x_1 + 0.9\cos x_1 u\end{aligned} \tag{40}$$

Remark 2. One advantage of descriptor models is that they sometimes can keep the natural structure of the dynamical model. Though it is easy to model a single inverted pendulum by differential equations, modeling an n-pendulum, i.e. the cascade of n pendulums one attached to each other, is highly difficult since all tangential forces need to be computed. Even if it is clearly possible to model the inverted pendulum of Fig. 9 as an ordinary state-space system, it is worth to highlight that modelling real systems in descriptor form gives the designer more options to achieve better performance. For this specific example, the mass acceleration can also be controlled with respect to the algebraic equation. In other words, in the ordinary state space model of the inverted pendulum system, pendulum acceleration is not an independent variable which can have an independent reference. But in the descriptor model, it can be controlled separately with its own reference and desired behavior. Furthermore, describing real systems by ordinary state-space models is sometimes very complicated if not impossible.

Consider the reference descriptor model (3) with matrices

$$E = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix},\quad A_d = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 3 & 2 & 1\end{bmatrix},\quad B_d = \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} \tag{41}$$

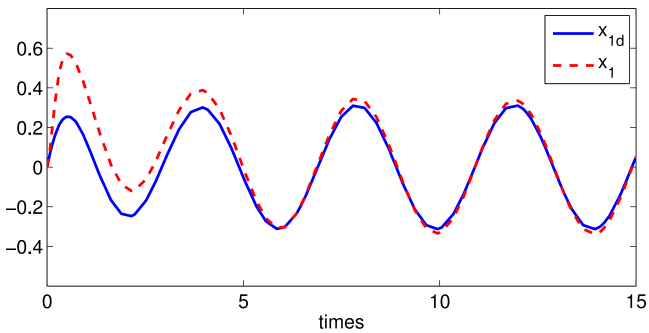

Fig. 10. Trajectory of the state variable x_1 and reference state variable x_{1d}.

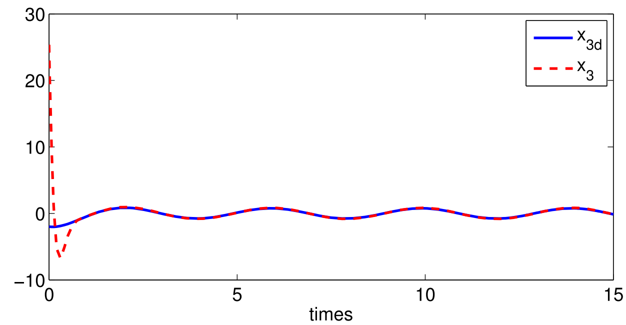

Fig. 12. Trajectory of the state variable x_3 and reference state variable x_{3d}.

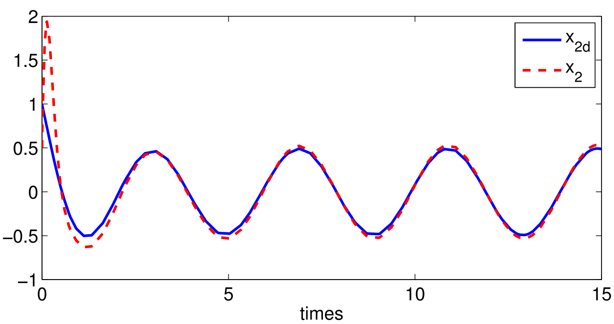

Fig. 11. Trajectory of the state variable x_2 and reference state variable x_{2d}.

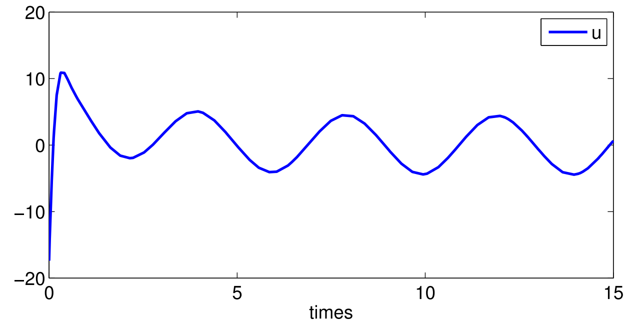

Fig. 13. Control input signal u.

and the reference input

$r(t) = \sin(0.5\pi t)$.

Five fuzzy sets, for each normalized state variable x_i, are characterized by the following membership functions

$$\mu^1(x_i) = 1 \ \ (1 + \exp(10(x_i - 0.4)))$$

$$\mu^2(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i - 0.2}{0.5}\right)^2\right)$$

$$\mu^3(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i}{0.5}\right)^2\right)$$

$$\mu^4(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i + 0.2}{0.5}\right)^2\right)$$

$$\mu^5(x_i) = 1/(1 + \exp(10(x_i + 0.4)))$$

In the simulations, the design parameters are taken as $\gamma_f = \gamma_g = 300$ and $\gamma_k = 10$. The initial state has been set to $\mathrm{x}(0) = [0, 1.5, 25]^T$ and the initial values of the fuzzy parameters $\boldsymbol{\theta}_f$ and θ_g have been set to 50. Solving (26) yields the following solution for matrix P:

$$\mathrm{P} = \begin{bmatrix} 1.63 & 0.36 & -0.16 \\ 0.36 & 0.73 & -0.43 \\ -0.16 & -0.43 & -0.06 \end{bmatrix}$$

.

Figs. 10–12 show that tracking performance is satisfactory and that the tracking error asymptotically vanishes. The control input signal is shown in Fig. 13. These results indicate that the developed controller can effectively solve the tracking control problem of descriptor systems with unknown parameters.

5. Conclusions

This paper has developed an indirect adaptive fuzzy control strategy for control-affine descriptor systems. The adaptation laws for the unknown descriptor system are provided via fuzzy logic approximation of unknown parameters to make the state track the desired response of a reference model when only the system order and the number of algebraic equations are known. Compared to existing adaptive control strategies developed for descriptor systems, the method proposed in this paper needs less information about the system under control and could be a convenient choice when modeling is a hard task or in the case that the system parameters are varying as unknown functions of time. In this paper, it is assumed that all states are available for measurement. Hence, it is worth to extend our study when there exist unmeasurable states. Moreover, possible future developments will concern direct adaptive fuzzy control strategies for nonlinear descriptor systems.

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